Outage Statistics in CDMA Mobile Radio Systems
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Abstract — The statistics of the outage events in the reverse link of a DS/CDMA wireless system are investigated taking into account the effects of correlated Rician fading. The outage probability and the average fade duration are derived by assuming a constant multipath intensity profile, which includes a direct-path component in the first-path. Simulations have been carried out which validate the analytical results.

Keywords — CDMA, fading channels, outage probability, average fade duration.

I. INTRODUCTION

Within the field of modern telecommunications, Code Division Multiple Access (CDMA) is one of the techniques which is gaining consideration for the development of mobile personal communications [1]. The Outage Probability (OP), i.e., the marginal probability that the Signal-to-Noise and Interference Ratio (SNIR) is below a threshold, is one of the most important QoS metric for CDMA wireless systems. Besides, knowledge of the Average Fade Duration (AFD) can be useful in defining coding and interleaving requirements. Recent contributions in the literature deeply investigate the outage event statistics in fading channels [2][3]. The goal of this contribution is to give an analytical model for the computation of OP and AFD on the reverse link of a wireless Direct Sequence CDMA system, characterized by a constant Multipath Intensity Profile (MIP) which includes a direct path component in the first path (i.e., Rician fading). An ideal open-loop power control is assumed, which compensates for the long-term variations due to path loss and shadowing experienced by the mobile user. Conversely, no sophisticated closed-loop mechanism able to compensate for small-scale multipath fading is supposed. On the other hand, closed-loop power control algorithms can be effective in compensating for rapid channel variations due to multipath fading only in cases where propagation and processing delays are small compared to the correlation time of the channel [4]. In an attempt to be thorough, we will assume that the multipath fading is mitigated at the base station through an ideal maximal ratio combiner (RAKE receiver).

II. OUTAGE PROBABILITY

Let L denote the number of resolvable paths of the frequency selective channel, $\alpha_{i,p}(t)$ the $p^{th}$ tap weight coefficient (relative to the $p^{th}$ multipath contribution) of the $i^{th}$ ($i = 0, N-1$) user channel and $\lambda_i$ the global multipath power of the $i^{th}$ user. It results

$$\lambda_i = \sum_{p=1}^{L} |\alpha_{i,p}|^2$$

Under the assumption of perfect long-term power control the average received Signal-to-Noise Ratio (SNR) is equal for all users and is given by

$$\gamma = \frac{1}{2N_n} PG \eta$$

where $PG$ is the processing gain of the spreading procedure, $N_n$ is the power of the background noise, and $\eta$ is the expected value of the multipath power, i.e., $\eta = E(\lambda_0) = \ldots = E(\lambda_{N-1})$. The SNIR experienced at the zeroth (desired) RAKE receiver, conditioned on the set of random processes $\lambda_i$, may be given as:

$$SNIR \{\lambda_i\}_{i=0,N-1} = \frac{\lambda_0 \gamma}{\eta + \frac{2\gamma}{3PG} \sum_{i=1}^{N-1} \lambda_i}$$

where the term $2/(3PG)$ derives from the cross-correlation properties of the spreading sequences [5]. The OP is defined as the probability that the output SNIR falls below a prescribed threshold $SNIR_t$. The goal is to compute this quantity averaged over multipath power fluctuations. If $F$ is the fading margin value, i.e. $\gamma = F \times SNIR_t$, the OP is given by

$$P_0 = Pr.(\lambda_0 < T)$$

where

$$T = \frac{1}{F} \left( \eta + \frac{2F \times SNIR_t}{3PG} \sum_{i=1}^{N-1} \lambda_i \right)$$

According to (4), $P_0$ is the cdf of $\lambda_0$ computed in $T$, $F_{\lambda_0}(T)$. In the presence of Rician fading, the received signal from a user may be thought as the superimposition of a direct component with many other resolvable scattered components. In the frequency selective fading channel considered here, the first multipath component of the $i^{th}$ user, referred to as $\alpha_{i,1}$, encompasses the direct and a portion of scattered components, while the remaining scattered components are temporary dispersed over $\alpha_{i,p}, p = 2,L$, according to the channel MIP. Under the assumption of uncorrelated scattering, the random processes $\alpha_{i,p}$ are modelled as independent complex Gaussian random processes with means

$$\begin{cases} E(\alpha_{i,p}) = \mu & i \in [0, N-1] \text{ and } p = 1 \\ E(\alpha_{i,p}) = 0 & i \in [0, N-1] \text{ and } p \in [2,L] \end{cases}$$

and variances

$$m_p = E[(\alpha_{i,p} - E(\alpha_{i,p}))^2] \quad i \in [0, N-1], p \in [1,L]$$

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Let $K_R = \frac{\sigma^2}{\eta}$ denote the Rice factor of the first multipath contribution which includes the Line of Sight (LOS) component. The evaluation of the OP in (4) requires a statistical characterisation of the threshold $T$. We observe that the randomness of $T$ depends on the sum of $N-1$ independent random processes $\lambda_i$ with mean $\eta$ and variance $(\sum_{p=1}^L m_p^2 + 2m_1 \mu^2)$. Thus, provided that the number of users $N$ is high, the central limit theorem allows to characterise the threshold $T$ as a Gaussian random process with mean and variance respectively given by

$$E(T) = \frac{\eta}{F} \left( 1 + \frac{2F \times SNIR_0 (N-1)}{3PG} \right)$$

(6)

$$\sigma^2_T(T) = \frac{N-1}{F^2} \left( \frac{2F \times SNIR_0}{3PG} \right)^2 \left( \sum_{p=1}^L m_p^2 + 2m_1 \mu^2 \right)$$

(7)

The evaluation of (4) may now be performed as

$$P_0 = \int_0^\infty \int_0^T f_T(T) f_{\lambda_0}(l) dldT = \int_0^\infty F_{\lambda_0}(T) f_T(T) dT$$

(8)

where $f_T(T)$ is the pdf of the Gaussian random process $T$. Under the assumption of constant MIP, i.e., $m_p = m, p = 1, L$, $F_{\lambda_0}(T)$ results the cdf of a non-central chi-square distribution with $2L$ degrees of freedom and non-centrality parameter $\mu^2 = K_R \theta m$, which can be numerically evaluated by means of a reasonable computational effort.

III. AVERAGE FADE DURATION

Parameter AFD is defined as the average time during which the decision variable $\lambda_0$ remains below the threshold level $T$. In this Section, we derive the AFD assuming that the interference level does not change over a burst error time, i.e., $T$ is constant during an outage event. The accuracy of this approximation will be verified in the next Section through simulation results.

In order to simplify the notations, denoting by $r_p = |\alpha_0 p|$, we get $\lambda_0 = \sum_{p=1}^L r_p^2$. Let us focus on the random process $r_p$. The tap weights coefficients $\alpha_0 p$ may be expressed as

$$\alpha_0 p = (X_p + \nu_p) + jY_p$$

(9)

where $X_p$ and $Y_p$ are Gaussian random processes with zero mean and variance $m/2$, $\nu_p$ is the average of the $p^{th}$ path, i.e., $\nu_p = \mu$, for $p = 1$ and $\nu_p = 0$, otherwise. The processes $X_p$ and $Y_p$ can be assumed to be stationary at least on the time scale of the fading variations. Thus, it is possible to define the correlation function

$$R_p(t_0) = E(X_p(t)X_p(t-t_0)) = E(Y_p(t)Y_p(t-t_0))$$

(10)

In a widely accepted model, the Gaussian processes are assumed to have a band-limited non-rational spectrum $[6]$ which corresponds to the correlation function

$$R_p(t_0) = \frac{m}{2} J_0 \left( 2\pi f_D |t_0| \right)$$

(11)

where $J_0(x)$ is the Bessel function of the first kind and of zeroth order and where $f_D = \nu / \lambda$ is the Doppler bandwidth, $\nu$ is the mobile speed, and $\lambda$ is the carrier wavelength. Let $\hat{r}_p$ denote the derivative of the envelope $r_p$ with respect to time. The joint cdf of the processes $r_p$ and $\hat{r}_p$ may be computed as in [6]

$$f_{r_p, \hat{r}_p}(r_p, \hat{r}_p) = \frac{2\pi m}{m} \left( \frac{2\pi \nu_p}{m} \right) e^{-\left( \frac{r_p^2 + \hat{r}_p^2}{m} \right)} \frac{1}{\sqrt{-2\pi \tilde{R}_p(0)}} \frac{1}{2\pi^{3/2}}$$

(12)

where $\tilde{R}_p(0)$ represents the double derivative of $R_p(t_0)$ evaluated in zero. Note that $r_p$ and its derivative are independent thus yielding

$$f_{\hat{r}_p}(r_p) = \frac{1}{\sqrt{-2\pi \tilde{R}_p(0)}} e^{\frac{-r_p^2}{2\tilde{R}_p(0)}}$$

(13)

The term $-\tilde{R}_p(0)$, which represents the variance of $\hat{r}_p$, may be given in a closed form: $-\tilde{R}_p(0) = m\pi^2 f_D^2$. The level crossing rate, $R(T)$, defined as the expected number of times per second the random process $\lambda_0$ crosses the threshold $T$ in the positive direction, is given by

$$R(T) = \int_0^\infty y f_{\lambda_0}(T, y) dy$$

(14)

where $f_{\lambda_0}(x, y)$ is the joint pdf of the process $\lambda_0$ and of its derivative $\lambda_0'$. From (13) and from the definitions of $\lambda_0$ and $\lambda_0'$, the random process $\lambda_0(\lambda_0 = z)$ is Gaussian distributed with zero mean and variance equal to $4m\pi^2 f_D^2$. Let us consider the normalised Gaussian random process $q = \left[ \lambda_0(\lambda_0 = z) \right] / \sqrt{z}$, which has zero mean and variance equal to $\delta^2 = 4m\pi^2 f_D^2$, and let us denote by $C_q(q_0)$ and $c_q(q_0)$ the cdf and the pdf of $q$, respectively. The joint cdf of $\lambda_0$ and its derivative can be obtained as

$$F_{\lambda_0}(x, y) = \int_0^x C_q \left( \frac{y}{\sqrt{x}} \right) f_{\lambda_0}(z) dz$$

(15)

Deriving expression (15) with respect to $x$ and $y$ we have

$$f_{\lambda_0}(x, y) = \frac{1}{\sqrt{x}} \frac{\partial}{\partial y} C_q \left( \frac{y}{\sqrt{x}} \right) f_{\lambda_0}(x)$$

(16)

It is now possible to evaluate the integral (14) as

$$R(T) = \int_0^\infty y f_{\lambda_0}(T, y) dy = \int_0^\infty \frac{\partial}{\partial T} C_q \left( \frac{y}{\sqrt{T}} \right) f_{\lambda_0}(T) dy$$

(17)

Making the substitution $\omega = \frac{y}{\sqrt{T}}$, it results

$$R(T) = f_{\lambda_0}(T) \sqrt{T} \int_0^{+\infty} \omega c_q(\omega) d\omega = f_{\lambda_0}(T) \sqrt{\frac{\delta^2}{2\pi}}$$

(18)
The average fade duration $G(T)$ conditioned on the threshold level $T$ can now be expressed as the ratio between the OP and the level crossing rate:

$$G(T) = \frac{F_{\lambda_0}(T)}{\gamma(T)}$$

(19)

Finally, an estimation of the average fade duration $G$ is given by:

$$G = \int_{0}^{+\infty} G(T)f_T(T)\,dT$$

(20)

IV. RESULTS AND COMPARISONS

In order to validate the analytical results derived in previous Sections, the proposed system has been simulated. In particular, a set of $L \cdot N$ discrete Gaussian processes with power spectra corresponding to the autocorrelation (11), has been generated by means of computer simulations. The sample time of the process, referred to as $H$, has been chosen so that the normalised cut frequency $f_D H$ of the Doppler digital filters verifies the condition $f_D H = 0.01$. The resulting Gaussian random processes have been properly combined to form the set of $N$ multipath powers $\lambda_i$, ($i=0, N-1$).

Finally, the OP and the normalised average fade duration $G/H$ have been estimated by comparing $\lambda_0$ with the threshold $T$ defined in (5). As for the analytical approach, it must be noticed that the integral expression (20) gives the average fade duration in seconds. However, the normalised average fade duration $G/H$ may be easily derived by substituting $f_D$ with $f_D H$ in (18). Moreover, equations (20) and (8), which cannot be expressed in a closed form, have been computed through numerical integration.

Comparisons between simulation results and the analytical approach are presented in Figs. 1 and 2 which show the behaviour of the outage probability $P_0$ and the normalised average fade duration $G/H$, respectively, versus the number $L$ of multiple paths and for different $K_R$ values. The following assumptions have been made: $SNR_0 = 7\,\text{dB}$, $F = 10$, $N = 10$, $PG = 150$. As shown in Figs. 1 and 2, the analytical approach well fits simulation results. The same degree of accuracy has been achieved for different environment conditions.

Let us focus on the conditioned AFD given in (19). The reduction in AFD with $L$ can be justified as follows: the greater $L$, the lower both the OP and the level crossing rate; however, the reduction with $L$ in the numerator in (19) is more appreciable than the decrease in the denominator. As expected, this effect is more evident for low $K_R$ values.

REFERENCES


